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ADAPTATION OF THE TH$\epsilon\mu$ FORMALISM FOR THE ANALYSIS OF THE EQUIVALENCE PRINCIPLE IN THE PRESENCE OF THE WEAK AND ELECTROWEAK INTERACTION

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16. ABSTRACT

The THεμ formalism, used in analyzing equivalence principle experiments of metric and nonmetric gravity theories, is adapted to the description of the electroweak interaction using the Weinberg-Salam unified SU(2) × U(1) model. The use of the THεμ formalism is thereby extended to the weak interactions, showing how the gravitational field affects $W_\mu^\pm$ and Z_μ^o boson propagation and the rates of interactions mediated by them. The possibility of a similar extension to the strong interactions via SU(5) grand unified theories is briefly discussed. Also, using the effects of the potentials on the baryon and lepton wave functions, the effects of gravity on transition rates are determined for β-decay, K-capture, and parity nonconserving transitions mediated in high-A atoms which are electromagnetically forbidden. Three possible experiments to test the equivalence principle in the presence of the weak interactions, which are technologically feasible, are then briefly outlined: (1) K-capture by the ^{55}Fe nucleus (counting the emitted X-rays); (2) forbidden absorption transitions in high-A atoms' vapor; and (3) counting the relative β-decay rates in a suitable α-β decay chain, assuming the strong interactions obey the equivalence principle. The report concludes with an outline of future work concerning (1) Eötvös tests involving the weak-interaction part of nuclear binding energies; (2) (β^-, γ, β^+) decays of nuclear isotope triplets and (β-γ) angular correlation experiments as weak-interaction equivalence tests; and (3) strong interaction tests.

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ADAPTATION OF THE $TH\epsilon\mu$ FORMALISM FOR THE ANALYSIS OF THE EQUIVALENCE PRINCIPLE IN THE PRESENCE OF THE WEAK AND ELECTROWEAK INTERACTION

I. INTRODUCTION

With the growth of interest in general relativity in the past 10 years, and a concomitant development of the technology to measure small physical effects, experimental gravitation has come into its own [1,2]. The experiments may be collected under four groupings: (1) astronomical, (2) laboratory, (3) gravitational wave detectors, and (4) space-borne. In the astronomical group are the old and famous observations of the precession of Mercury's perihelion [3] and the bending of starlight by the Sun [4], as well as the modern radar/time-delay experiments [5] and solar light-bending observations using radio waves [6]. One classical laboratory example is the Pound-Rebka experiment [7] involving the redshift of gamma rays in the Earth's gravity. The other is the Eötvös-Braginsky-Dicke experiments [8] which study whether objects of different internal compositions and/or structures fall at the same rates in the gravitational field. Since 1956, gravitational wave detectors have been pioneered by Weber [9] and developed to a large degree in recent years [10]. There is controversy concerning the results, but work is continuing on newer, more sensitive detectors [11], some of it very ingenious [12-14]. Space-borne experiments offer new opportunities and challenges. The classic example of these has been the Harvard-MSFC rocket-borne hydrogen maser clock experiment [15], measuring the gravitational redshift to high accuracy. Another type of example involves tracking of spacecraft to measure signal time delays [16] and other effects, such as rotational frame dragging [17] and gravitational radiation detection [18]. An attempt to measure the gravitational induction field in the precession of the spin of a gyroscope in the presence of the Earth's angular momentum, the Stanford-MSFC gyroscope experiment, will be the first of the new Shuttle-borne space experiments [19].

This burgeoning of experimental work has spurred theorists to develop appropriate tools for the analysis of experiments and the determination of their implications for whichever currently viable gravitational theories have been proposed [20]. Such a formalism was developed for the study of gravitational theories which can be put in a metric form (gravitational potentials are in metric tensor components) by Will and Ni [21] and by Nordvedt [22]. A method of studying all gravity theories was then developed by Thorne et al. [23]. They classified gravitational theories by the mathematical structures peculiar to each and by the way different statements of the equivalence principle are incorporated into the theory. The subsequent necessity to find a formalism to include nonmetric theories in these studies was first met by Lightman and Lee [24] in studies of electrostatic energy contributions to the structures of nuclei falling freely in gravitational fields (in Eötvös experiments).

Will [25] subsequently adapted the $TH\epsilon\mu$ formalism to analysis of the hydrogen maser redshift experiment, studying magnetic dipole transitions in an electrostatic background. The formalism has been extended by Haugan and Will [26] to analyze magnetostatic energy contributions to nuclear structure in Eötvös experiments. Thus far, the formalism has been used to study the equivalence principle in the presence of the following interactions and to the following order in the gravitational potential $U \sim m/r$: electrostatic at first order [24]; electrostatic, magnetic-dipole electromagnetic transitions, and electromagnetic propagation at first order [25]; and magnetostatic at second order [26].

It is of interest to extend this formalism to other electromagnetic tests, such as to its verification for the quadrupole transitions in nuclei (Pound-Rebka experiment [27]) and to a magnetostatic test at first order in U, using nuclear magnetic resonance and including magnetostatic, spin-spin, and electron paramagnetic and diamagnetic effects [28]. These and other possible electromagnetic experiments are, however, only concerned with one fundamental force. It would be of interest to carry out such experiments for the other fundamental forces, the strong and weak interactions.

This interest arises from the classification of equivalence principles as discussed by Thorne et al. [23]. The three classes are: the weak equivalence principle (WEP), the Einstein equivalence principle (EEP), and the strong equivalence principle (SEP). The WEP states that a particle world line is independent of its structure and composition. The EEP states that WEP is true and, further, that the outcome of any local nongravitational experiment or process is independent of its spacetime location and its apparatus' velocity in free fall. Finally, the SEP states also that WEP is valid and, further, that the outcome of any local test experiment (including gravitational) is independent of spacetime location or velocity in free fall. Now, since Einstein's equations state $R_{\mu\nu} - 1/2\ g_{\mu\nu}\ R = \kappa\ T_{\mu\nu}$ for any $T_{\mu\nu}$ regardless of its underlying field, the universality of κ requires that experimental results produced by different interactions be independent of the type of interaction. This was first pointed out by Brecher [29]. Thus, if clock-rate experiments for two clocks, "unwinding" or "ticking away" by different interactions taking place within them, produce two different variations within the same varying gravitational fields, then the universality of κ is violated. The coupling of the stress-energy tensor $T_{\mu\nu}$ to the Einstein tensor $R_{\mu\nu} - 1/2\ g_{\mu\nu}\ R$ would vary with the nature of the interaction producing $T_{\mu\nu}$, and, thus, the outcome of gravitational experiments would vary. Hence, SEP would be violated.

The experimental analyses previously cited [24-29], while dealing with different electromagnetic interactions, still derive from one Lagrangian density, that of electromagnetism (albeit different "pieces" of that Lagrangian in each case). Their stress-energy tensor is of the same form for all of them. To test SEP it would be better to test interactions with different Lagrangian densities, i.e., to test forces other than electromagnetism. A natural candidate is the weak interaction.

Among relativity experiments, two types stand out: (1) Eötvös (free-fall) experiments and (2) clock experiments. Clock experiments may be further divided into three types [30]: (1) ruler clocks, based on a cavity's length of resonant frequency variation in a varying gravitational potential (such as the hydrogen maser clock); (2) oscillator clocks, based on a vibration rate's variation (such as the Pound-Rebka experiment); and (3) decay clocks, based on variations in the decay rate of unstable (or excited) systems. The weak interaction offers us a clock of the third type in β-decay or its related processes such as K-capture [31]. Further, a formalism exists for treating the weak interaction as an extension of electromagnetism: The Weinberg-Salam unified electroweak theory [32].

A preliminary approach to Eötvös-type experiments involving nuclear weak interaction energies has been presented by Haugan and Will [33] and extended by Hsu [34] to include nuclear weak self-energies. These results are tentative (or incomplete).

The remainder of this report is organized as follows: Section II reviews the TH$\epsilon\mu$ formalism for electromagnetism in gravitational fields and the results for Eötvös and redshift experiments. The Weinberg-Salam model is outlined in Section III, and the motivating argument is then given for the extension of the TH$\epsilon\mu$ formalism to the electroweak interactions. (Basically, it is that the potentials ϵ and μ have an effect on $W^{\pm}_{\mu}$ and Z_{μ} boson propagation in a manner similar to their effect on A_{μ} photon propagation.) The extension is carried out in Section IV, which concludes with a brief discussion of a possible extension of the formalism to the strong interactions via the SU(5) grand-unified gluon models. In Section V the effects of T and H on baryon and lepton wave functions (also possible ϵ and μ effects) and propagations are determined. Section VI shows the effects of gravity on: transition rates for β-decay and K-capture, and parity-nonconserving transitions mediated in high-A atoms by Z_{μ}-boson exchange. Section VII contains a description of three possible experiments to which these results may be applied to test the equivalence principle in the presence of the weak interactions. Their feasibility and practicality are discussed. The conclusion, Section VIII, outlines future work: (1) theoretical investigations of Eötvös-type experiments for nuclear weak interaction binding energies, up to weak magnetostatic (equivalent to the post-post-Coulombian analysis of magnetostatics by Haugan and Will [26]); (2) (β^-, γ, β^+) nuclear isotopic triplet decays to study gravitational effects on dynamical weak magnetism; and (3) future strong interaction tests.

II. The TH$\epsilon\mu$ FORMALISM

The TH$\epsilon\mu$ formalism was motivated by a desire to transform nonmetric gravitational theories into a metric form for comparison with equivalence principle experiments [24]. An analysis of a theory of gravitational

theories [23], metric and nonmetric, made this desirable. The need for such a formalism is further aggravated by the fact that, even in the simplest free-fall and clock experiments, the principle of equivalence is violated by nonmetric theories, as pointed out by Nordvedt [35].

Following Lightman and Lee [24], we start with the simplest relativistic Lagrangian for particle motion:

$$\mathcal{L} = \int \left[- m_o \, (1-v^2)^{1/2} + e \, v^\mu \, A_\mu \right] dt \tag{1}$$

where m_o, v^μ, and e are the particle rest mass, velocity and charge, and A_μ is the electromagnetic field. If we take the speed of light, c = 1, it is clear that the expression $(1 - v^2)^{1/2}$ in equation (1) is just ds, the line interval in special relativity, if we multiply by dt and get ds = $c^2\, dt^2 - v^2\, dt^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu$. The principle of minimal coupling then substitutes the general metric $g_{\mu\nu}$ for the Lorentz metric $\eta_{\mu\nu}$. For an isotropic metric, equation (1) is then

$$\mathcal{L} = \int \left[- m_o \, (T - Hv^2)^{1/2} + ev^\mu \, A_\mu \right] dt \tag{2}$$

where $g_{oo} = T$ and $g_{ij} = H\delta_{ij}$. If the gravitational field is then interpreted as providing a refractive medium with permittivity ε and permeability μ different from their vacuum values, gravitational effects on the electric and magnetic fields are then directly contained in Maxwell's equations:

$$\vec{\nabla} \cdot (\varepsilon\vec{E}) = 4\pi\rho \tag{3}$$

$$\vec{\nabla} \times (\vec{B}/\mu) = 4\pi \, \vec{J} + \frac{\partial(\varepsilon\vec{E})}{\partial t} \tag{4}$$

where $\vec{E} = \vec{\nabla}A_o - \partial\vec{A}/\partial t$ and $\vec{B} = \vec{\nabla} \times \vec{A}$. This formalism was previously developed to examine the passage of electromagnetic radiation through gravitational fields by Plebanski [36] and Volkov et al. [37] and has been used extensively by Mashhoon [38] to consider a number of variations of that problem. The $TH\varepsilon\mu$ formalism is a specialization of those papers.

One then varies the Lagrangian to obtain equations of motion, expanding in powers of T and H and their derivatives with respect to the Newtonian gravitational potential U, the gravitational acceleration $\vec{g}$, the particle velocity $\vec{V}$, and combinations of them to whatever order is desired. The particle acceleration is then a function of those quantities and of the Lorentz acceleration $\vec{A}_L$ with

$$\vec{A}_L = \frac{e}{m} [- \vec{\nabla} A_o + \vec{\nabla} (\vec{V} \cdot \vec{A}) - d\vec{A}/dt]. \tag{5}$$

Electrostatic equivalence principle tests take the expansion to $O(v^2)$, as in Lightman and Lee [24], while magnetostatic tests involving free fall of nuclei must take the expansion to $O(gv^4)$, as in Haugan and Will [26]. Alternatively, one can construct a Hamiltonian from equation (2) and use this to derive and solve Dirac's equation. Will has done this [25] and applied it to atomic hydrogen to analyze the hydrogen maser clock experiment, thus extending the use of the $TH\varepsilon\mu$ formalism to quantum mechanics.

When the formal computations are done, the quantities T, H, ε, and μ are expanded as a power series in the Newtonian potential U:

$$T = 1 - 2\alpha U + 2BU^2 + \ldots . \tag{6a}$$

$$H = 1 + 2\gamma U + \frac{3}{2} SU^2 + \ldots . \tag{6b}$$

$$\varepsilon = 1 + \varepsilon_1 U + \varepsilon_2 U^2 + \ldots . \tag{6c}$$

$$\mu = 1 + \mu_1 U + \mu_2 U^2 + \ldots . \tag{6d}$$

Lightman and Lee [24] have shown (compare Plebanski [36], Volkov et al. [37], and Mashhoon [38]) that the condition for the theory to be metric and the equivalence principle to hold is $\varepsilon = \mu = (H/T)^{1/2}$ exactly. To keep this evident the power expansions of equation (6) are rewritten, inserting an appropriate expansion of $(H/T)^{1/2}$ from equations (6a) and (6b) into equations (6c) and (6d), as done by Haugan and Will [26]. (T and H are in the form of the familiar PPN expansion.) They obtain

$$\varepsilon = 1 + (a_1 - \Gamma_o) U + (a_2 - \Gamma_1) U^2 + \ldots . \tag{7a}$$

$$\mu = 1 + (a_1 - \Lambda_o) U + (a_2 - \Lambda_1) U^2 + \ldots . \tag{7b}$$

where $a_1 = 1 + \gamma$ and $a_2 = \frac{1}{2}(3 + 2\gamma - 2\beta - \gamma^2 + \frac{3}{2}\delta)$. All the Γ_i and Λ_i coefficients vanish for metric theories but not for nonmetric ones. Equivalence principle experiments thus test for the existence of these nonmetric Γ_i (electric) and Λ_i (magnetic) coefficients.

The quantum-mechanical considerations of Will [25] for the redshift experiments required no expansion of T, H, ϵ, or μ to test the metric meshing law. The results are exact. However, to illustrate at what order a theory might be perceived to have violated the equivalence principle, the expansions were carried out. At first order, the principle and fine contributions to the frequencies measure Γ_0, while a "magnetic" parameter Υ appears in the hyperfine magnetic-dipole transition, as expected. At second order in U things get mixed together (ϵ and μ). That is expected since the experiment [26] deals with electromagnetic radiation and not dc electric or magnetic fields alone.

Haugan [39] has completed a more comprehensive study of the relation of the equivalence principle to the conservation of energy, elucidating the role of the TH$\epsilon\mu$ formalism more fully. There the emphasis is on the role of the formalism in analyzing the effects of electromagnetic structure on free fall and the equivalence principle. We will go on from that to adapt the formalism to analysis of weak interaction structure. The unified gauge field models of the weak and electromagnetic (electroweak) interactions provide a convenient formalism to extend the TH$\epsilon\mu$ formalism from electromagnetism to the weak interactions (i.e., from a part of the electroweak interaction to all aspects of the interactions).

This section concludes with the example of quantum electrodynamics to illustrate the procedure to be followed. The text of Bjorken and Drell [40] is the main source for our computations. Write down the amplitude for the process in question, inserting the TH$\epsilon\mu$ factors from the classical equations. Where confusion arises, follow the procedure outlined in the lectures of Brodsky [41], recover the Dirac equation and keeping step-by-step track of the TH$\epsilon\mu$ factors. It is then a simple process to extract them from the Dirac (or Maxwell) equations and return forward through the calculations back to the original Q.E.D. scattering amplitude. In this way the effects of the TH$\epsilon\mu$ potentials on the rates of such processes as electron-proton Compton scattering (as in Reference 40) or atomic absorption and emission (as in Reference 41) can be determined.

III. THE WEINBERG-SALAM MODEL AND A MOTIVATIONAL ARGUMENT

The Weinberg-Salam Model gives a complete description of the weak interactions, unified with the electromagnetic interactions. It is briefly outlined here, mainly for the sake of those relativists and the readers unfamiliar with the model. However, the outline given here is no substitute for detailed reading of the literature on the subject. The reader is

urged to consult the previously cited references [32] as well as the excellent reviews of Abers and Lee [42] and the recent Nobel lectures of Weinberg, Salam, and Glashow [43].

The gauge symmetry chosen is $SU(2) \times U(1)$, giving three gauge mesons a^i_μ associated with $SU(2)$ and one b_μ with $U(1)$. The Lagrangian is then written in three parts:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{leptons} + \mathcal{L}_{scalars} \tag{8}$$

containing gauge boson, lepton, and scalar combinations. The gauge field contribution contains the usual squared field strengths of vector field theories:

$$\mathcal{L}_{gauge} = -\frac{1}{4} A^i_{\mu\nu} A^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{9}$$

where

$$A^i_{\mu\nu} = \partial_\mu a^i_\nu - \partial_\nu a^i_\mu + g\, \varepsilon^i_{jk}\, a^j_\mu\, a^k_\nu \tag{10a}$$

$$B_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu \tag{10b}$$

The lepton spinor fields contain a right-handed singlet R and a left-handed doublet L; so then the Lagrangian contribution is

$$\mathcal{L}_{leptons} = \bar{R} i\gamma^\mu(\partial_\mu + ig'b_\mu)R + \bar{L} i\gamma^\mu \left(\partial_\mu + \frac{i}{2} g'b_\mu - \frac{i\tau^i}{2} g a_{i\mu}\right)L \tag{11}$$

where g and g' are coupling constants, and τ^i and γ^μ are the Pauli and Dirac matrices. For the scalar $SU(2)$ doublet ϕ, we have

$$\mathcal{L}_{scalars} = \left(\partial_\mu \phi^+ + \frac{ig'}{2} b_\mu \phi^+ + \frac{ig}{2} \tau^i a_{i\mu} \phi^+\right)\left(\partial^\mu \phi - \frac{ig'}{2} b^\mu \phi - \frac{ig}{2} \tau^i a_i^{\ \mu} \phi\right) - V(\phi^+ \phi) \tag{12}$$

where the form for the potential V is

$$V = m^2 \phi^+ \phi + \lambda(\phi^+ \phi)^2 \tag{13}$$

with m and λ representing self-couplings.

One then sets $m^2 < 0$ so that one component of the scalar doublet develops a nonzero vacuum expectation value, written as

$$<\phi> = \begin{pmatrix} o \\ v \end{pmatrix} / \sqrt{2}. \tag{14}$$

This breaks the SU(2) and U(') symmetries. The value is chosen real $V = [-m^2/\lambda]^{1/2}$ and the scalar fields are redefined in terms of the broken generators of the symmetry groups. An interaction term between the scalars and leptons is added to the Lagrangian:

$$\mathcal{L}_{int} = - G[\bar{R} \phi^+ L + \bar{L} \phi R]. \tag{15}$$

One then carries out the full set of U-gauge transformations to reveal the particle content of the model. Up to that point, the particles were all massless; the symmetry breaking, set of U-gauge transformations applied, and interaction with the scalar field's vacuum expectation value endows the leptons and gauge bosons with a mass spectrum. The mass spectrum shows in the classical field equations for each particle; this is the Higgs mechanism. The transformation is of the form

$$U = \exp(- i \xi_i \cdot \tau^i/2v) . \tag{16}$$

The reader can check the references for the details.

In the resulting Lagrangian, the ϕ field, contained in the ξ_i, vanishes with its vacuum expectation value contributing to the particle masses. One finds an electron mass

$$m_e = Gv/\sqrt{2} \tag{17a}$$

with the neutrino mass still zero since only the electron has both left-

and right hand representations. There appears a charged $W_\mu^\pm$ boson field of

$$W_\mu^\pm = (a_\mu^1 + i a_\mu^2) / \sqrt{2} \tag{17b}$$

with mass

$$M_W = \frac{1}{2} g v \ . \tag{17c}$$

Two neutral fields appear, the Z_μ:

$$Z_\mu = \frac{g' b_\mu - g a_\mu^3}{\sqrt{g^2 + g'^2}} \tag{17d}$$

of mass

$$M_Z = \frac{1}{2} v (g'^2 + g^2)^{1/2} \tag{17e}$$

and the A_μ:

$$A_\mu = \frac{g b_\mu + g' a_\mu^3}{\sqrt{g^2 + g'^2}} \tag{17f}$$

of mass zero (the photon). To include hadrons, new terms are added to the Lagrangian. Their masses are also generated by $\langle\phi\rangle$.

After quantization and renormalization are complete, one is prepared to carry out computation of the amplitude for processes, of which the most important one to be considered here is the beta decay shown in Figure 1. There are numerous correction diagrams to Figure 1, but the only essential one is shown here. Gravitational effects on higher order diagrams may be inserted by applying the discussion here to the

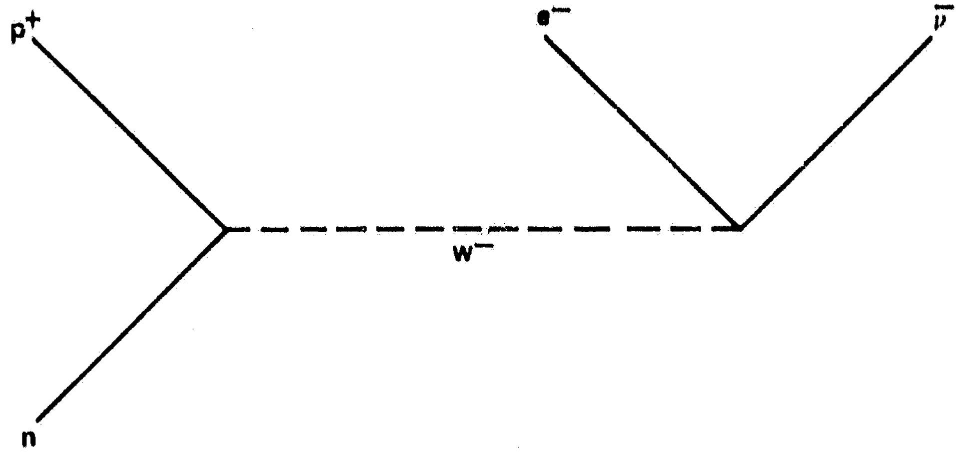

Figure 1. Usual Feynman graph depicting the process of neutron (n) decay into proton (p^+), electron (e^-), and antineutrino ($\bar{\nu}$) via emission of a W^- boson.

appropriate references, as in Reference 44. The amplitude in Figure 2 involves just the neutron and proton spinors, the W^- boson propagator, and the electron and antineutrino spinors. One wishes for a good quantum theory of gravity to model gravitational effects in a diagram such as Figure 1. Then we would have one external graviton line, probably at the vertices and at the W^- propagator, plus possibly several internal graviton lines (i.e., there would be several diagrams to consider). This procedure is still uncertain at best. We rather model gravitational effects in a phenomenological way here.

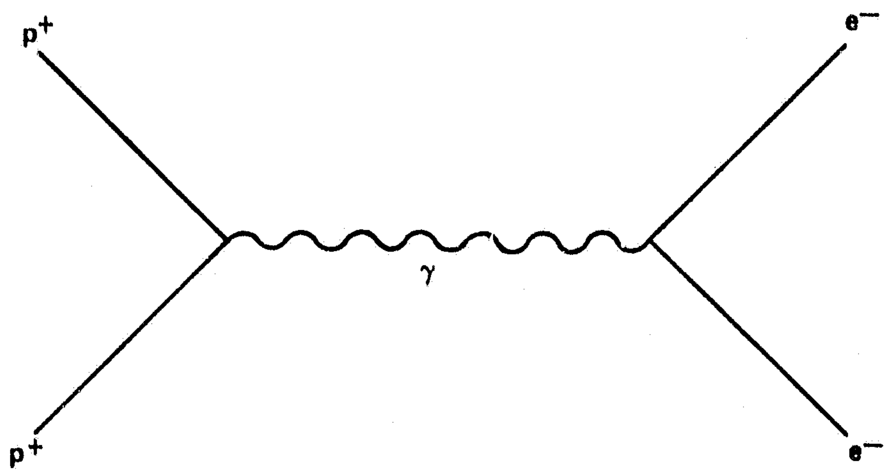

Figure 2. Graph for electron (e^-) - proton (p^+) scattering by photon (γ) exchange.

Consider the diagram in Figure 2. It is the proton-electron scattering diagram which also represents the exchange of a photon γ by electron and proton in the bound state of the hydrogen atom [41]. We may view the process as taking place in a medium in which the photon's propagation is affected by refraction. This corresponds to absorption and re-emission by a dipole, as shown in Figure 3. Now Figure 3 is easily inserted into Figure 2 to give Figure 4, thus showing the effects of the presence of the refracting medium on the propagation of the photon between p^+ and e^- and, hence, the effect of the presence of the medium on the process of scattering or on the change of state of an atom. Figure 4, of course, gives the effects of vacuum polarization on the process in question. Now consider the following: The usual electric permittivity ϵ and magnetic permeability μ represent the effect of the vacuum on the field strengths $F_{\mu\nu}$ propagating in that vacuum. They arise because vacuum polarization provides an index of refraction n_o to the propogation of virtual or real photons through the vacuum sea of virtual dipoles. (If vacuum polarization did not exist, the speed of light would be infinite and electromagnetism could be described by action-at-a-distance theories!) This is nothing new. It is well known how ϵ and μ have been calculated from the oscillating dipole interacting with the propagating vector potential in a refractive medium.

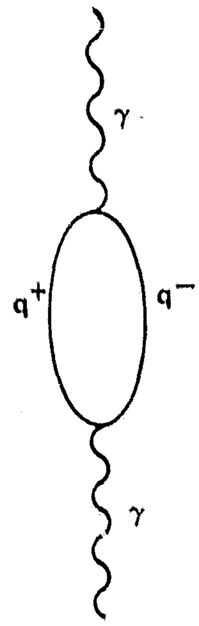

Figure 3. Vacuum polarization (or dipole absorption and reemission) graph for photon (γ) via two opposite charges q^+ and q^- which may be leptons or quarks.

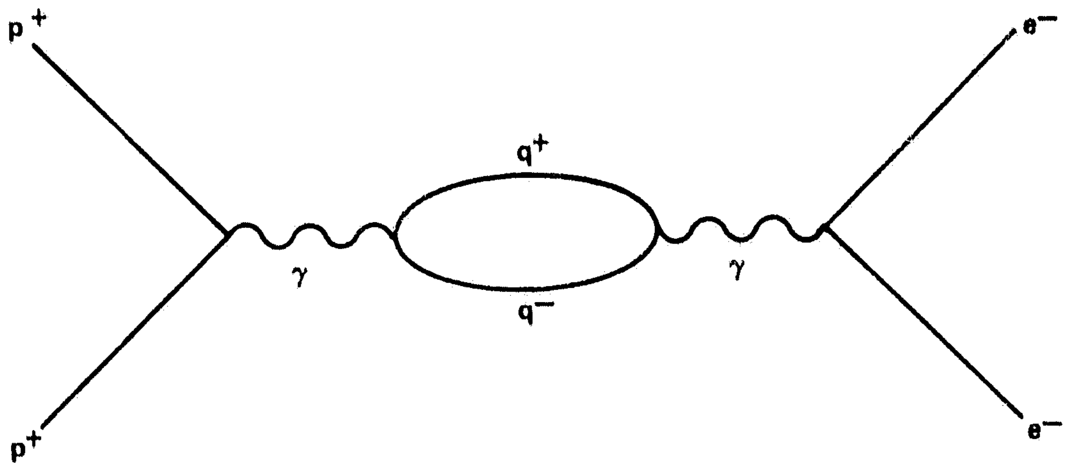

Figure 4. Vacuum polarization contribution to electron-proton scattering.

The THεμ formalism provides a way of phenomenologically including gravity in electromagnetic interactions as a refractive effect on real and virtual photon propagation, as schematically shown in Figure 5. This may also be inserted into the scattering diagram, as in Figure 6, where the contributions of T and H are indicated also. We now show why it may be extended to the weak interactions.

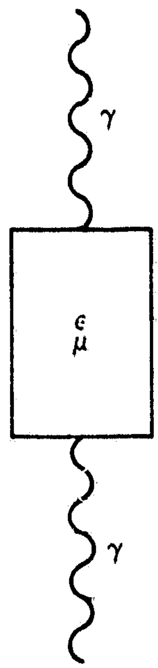

Figure 5. Gravitation "refractive-index" effects on photon propagation as a "black box" in which ε and μ interact with photon (or act on it), effectively replacing the dipole in Figure 3.

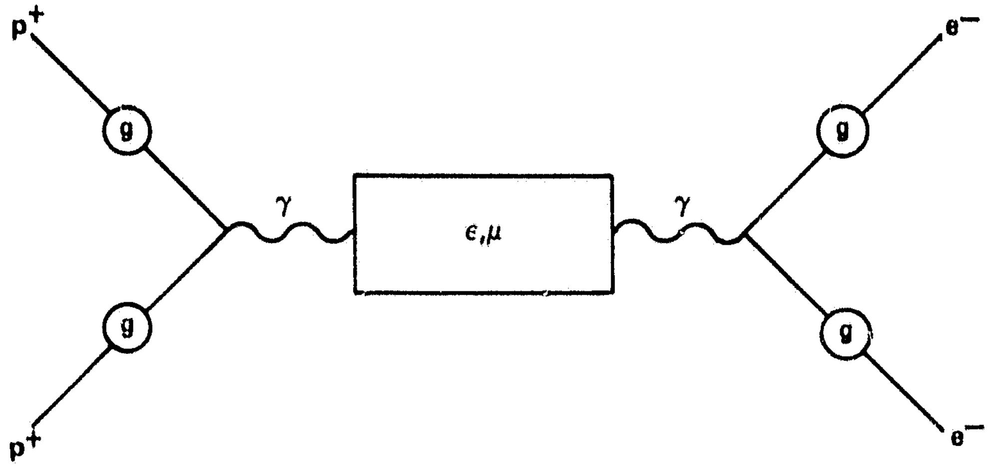

Figure 6. Alteration of the graph in Figure 4, indicating "black boxes" where ϵ and μ should interact with the photon and circles where $TH\epsilon\mu$ act on the lepton and hadron spinors.

In the Weinberg-Salam model, the photon A_μ is a linear combination of two gauge bosons, a_μ^3 and b_μ, as is the Z_μ boson. It makes little sense to assume that if the photon propagates through a gravitational field only the b_μ is affected and not the a_μ^3. Both a_μ^3 and b_μ must be affected the same way by the gravitational artificial index of refraction. Since a_μ^3 is a part of an isotopic triplet, its confederate bosons a_μ^1 and a_μ^2, by the same argument, should be affected the same way by gravity as the b_μ. Therefore, the ϵ and μ potentials should affect A_μ, Z_μ, and $W_\mu^\pm$ propagation in the same way. One can imagine a vacuum-polarization graph in Figure 7 being replaced as in Figure 8. The process of most interest will be K-capture by the Fe^{55} nucleus. Its graph, with gravitational factors entered schematically, is given in Figure 9. (The amplitude is the same as for Figure 8.)

The reader should be careful to keep in mind that this report is talking about coupling these interactions to gravity. Someone may at this point think that it has been discussing a naive and stupid argument coupling the Z_μ to electric currents, which is obviously not its intention. The motivational argument is now almost complete.

This discussion may not have seemed necessary to those physicists who truly believe that: (1) the electromagnetic and weak interactions are now truly unified and/or (2) that the universality of gravitational coupling and the principle of equivalence imply that gravitation couples

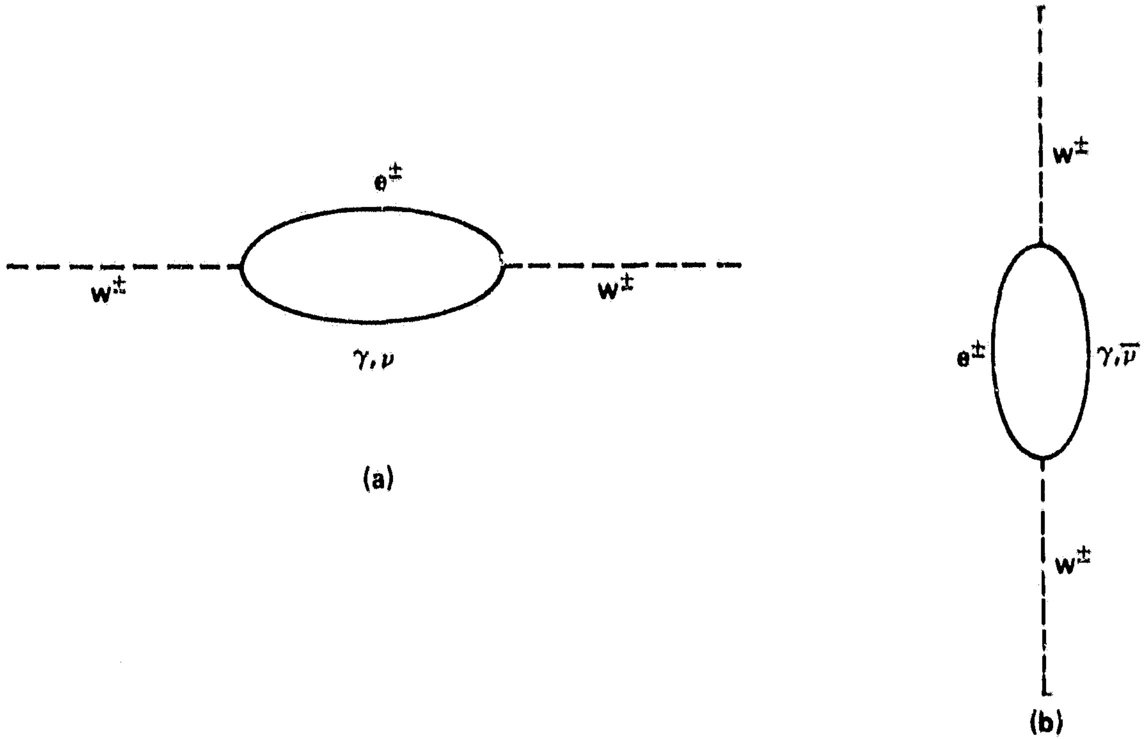

Figure 7. One of many possible vacuum polarization graphs on (a) virtual and (b) real, $W^{\pm}$ boson propagation.

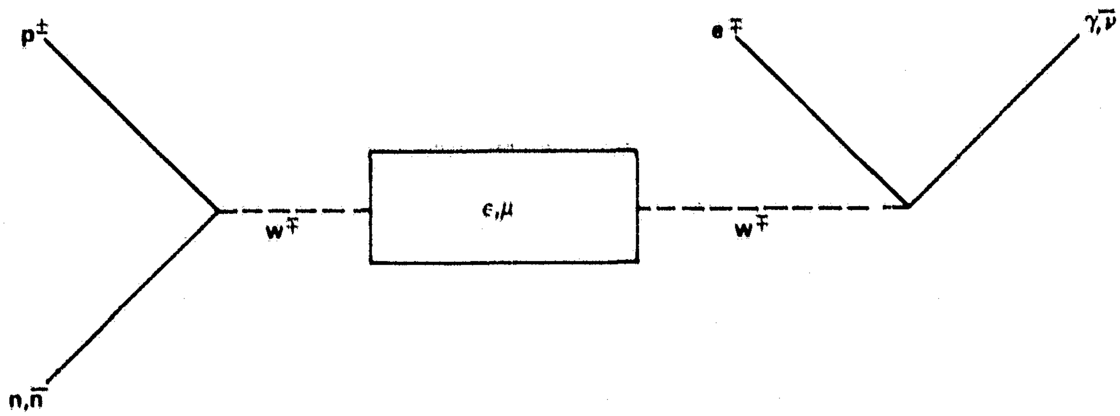

Figure 8. Beta-decay graph of Fig. 1. with effect of ε - μ black box inserted for W^{-} boson propagation.

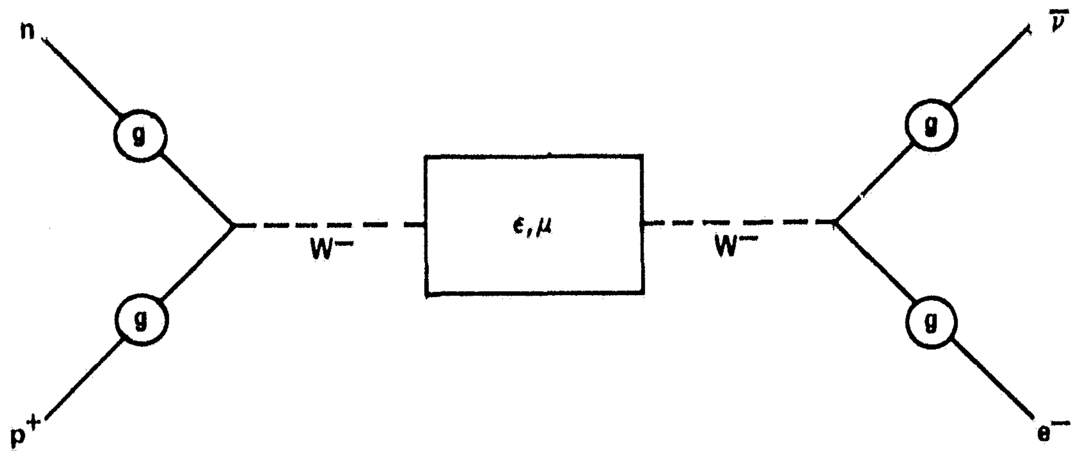

Figure 9. Electron K-capture graph with ε - μ "black-box" interaction and circles indicating TH$\varepsilon\mu$ effect on nucleons and leptons.

to all vector theories in the same way. But no experiment has tested this. This report proceeds to suggest some experiments to test this, assuming that (1) and (2) above are true. What should be done if theoretical prediction were contradicted by experimental outcome? Clearly, an experimental test is desirable. The motivational argument, while long, is a clear derivation; derivation is preferable to assumption.

IV. EXTENSION OF THE TH$\varepsilon\mu$ FORMALISM TO THE WEAK INTERACTIONS

The actual form of the electromagnetic Lagrangian when ε and μ are properly inserted is

$$\mathcal{L}_{el} = -\frac{1}{4}\left[\varepsilon F_{oi} F^{oi} + \mu^{-1} F_{ij} F^{ij}\right], \tag{18}$$

while for the electron and the photon-electron interaction it is

$$\bar{e}\, i\, \gamma^{\mu} (D_{\mu} + i q A_{\mu} + m)\, e \tag{19}$$

where q and m are the charge and mass and D_{μ} is the spinor covariant derivative

$$D_\mu = \partial_i + \Gamma_i \ . \qquad (20)$$

For a metric of the form

$$ds^2 = Tdt^2 - H\delta_{ab}\, dx^a\, dx^b, \qquad (21)$$

equation (19) can eventually be brought into the form [25]

$$\hat{H}e = \{T^{1/2}\, [m\beta + H^{-1/2}\, \alpha^i\, (\partial_i - qA_i)] - qA_o\}\, e \qquad (22)$$

Where $\beta = T^{-1/2}\gamma_o$ and $\alpha_i = -H^{-1/2}\, \beta^{-1}\, \gamma_i$, after some coordinate transformations and rescaling of the spinor fields e.

If we now define

$$Z_{\mu\nu} = \partial_\mu\, Z_\nu - \partial_\nu\, Z_\mu \qquad (23a)$$

$$F_{\mu\nu} = \partial_\mu\, A_\nu - \partial_\nu\, A_\mu \qquad (23b)$$

and

$$W^\pm_{\mu\nu} = \partial_\mu\, W^\pm_\nu - \partial_\nu\, W^\pm_\mu \ , \qquad (23c)$$

we may write the gauge boson Lagrangian as

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\, [Z_{\mu\nu}\, Z^{\mu\nu} + F_{\mu\nu}\, F^{\mu\nu} + 2W^+_{\mu\nu}\, W^{-\mu\nu}\ ] \qquad (24)$$

This will give the equation of motion for the free propagation. We insert $\varepsilon/2$ and $1/2\mu$ into the electric field strength and magnetic field strength parts of the Lagrangian as in the purely photon Lagrangian in equation (18). For the mass terms, we look back at the $a_{\mu\nu}$ and $b_{\mu\nu}$ Lagrangian with the scalars which provide the mass terms. The effect on the m_Z^2 and m_W^2 terms is negligible, if any (in only the phase of the Z_μ and $W^\pm_\mu$).

The lepton Lagrangian takes the form

$$\bar{\ell}\, i\gamma^{\mu}\left[D_{\mu} - i\left(\frac{g}{\sqrt{2}}W_{\mu} - \frac{(g^{2}+g'^{2})^{1/2}}{2}\, Z_{\mu}\right)\right]\ell \tag{25}$$

where ℓ is a lepton spinor field. If we wish to include the photon as a U(1) gauge boson field, the derivative D_{μ} is transformed to

$$D_{\mu} \to D_{\mu} - ieA_{\mu} = \Delta_{\mu}\ , \tag{26}$$

thus, including the effects of electromagnetism on the $W^{\pm}_{\mu}$ and Z_{μ} processes. We then obtain a Dirac equation for the spinors:

$$\dot{H}\,\ell = \left\{T^{1/2}\left[m\beta + H^{-1/2}\alpha^{i}\Delta_{i} - \frac{g}{\sqrt{2}}W_{i} + \frac{(g^{2}+g^{1\,2})^{1/2}}{2}Z_{i} - \right.\right.$$

$$\left. - qA_{o} - \frac{g}{\sqrt{2}}W_{o} + \frac{(g^{2}+g^{1\,2})}{2}Z_{o}\right\}\ell . \tag{27}$$

The fields A_{μ}, $W^{\pm}_{\mu}$, and Z_{μ} in equation (27) are determined from the classical solutions of their field equations. The effects of the $TH\epsilon\mu$ potentials on the nucleus and leptons can then be read out of equation (27). Thus the extension is accomplished.

It should be possible to use the same argument to extend the formalism to the strong interactions, at least in the quark-vector gluon model in grand unified SU(5) or similar theories. Consider that in the very early high-momentum stage of the universe all coupling constants became equal and gauge symmetries are restored. Then all interactions have the same strength and are indistinguishable. During that stage ϵ and μ should appear in their Lagrangian the same way for each of the fields. As the universe expands, symmetries are broken and coupling constant values diverge from each other, establishing a separation of the various forces. However, since the gravitational potentials are already included in the unified Lagrangian, they remain in the separated Lagrangians. From this point of view, one could say that the principle of equivalence might be a consequence of the equivalence of all interactions in the early universe. It is not clear how the formalism would be extended

to the nucleon-pion form of the strong interaction. At any rate, such an extension is properly the subject of another report and will be treated in a search for strong-interaction experiments to test the equivalence principle.

V. EFFECTS OF THE GRAVITATIONAL POTENTIALS ON THE SPINOR AND BOSON FIELDS

The exact behavior of the spinors and vectors depends on the choice of interaction, i.e., on what fields are present. When the particular fields and the modes they are in are specified, determination of the effects of T, H, ε, and μ is greatly facilitated by reference to the published calculations in References 24 through 28 and 34. It is a matter of inserting the coefficients properly in the initial equation and then carefully keeping track of them through to the solution.

Evaluating these effects for equations (18) through (27), we have: For the spinors

$$\psi \to \psi \cdot \left(\frac{H}{\varepsilon\, T^{1/2}}\right), \tag{28}$$

for the gauge bosons

$$Z_\mu \to Z_m \cdot \mu,\ Z_o/\varepsilon \tag{29a}$$

$$W_\mu \to W_m \cdot \mu,\ W_o/\varepsilon \tag{29b}$$

and

$$A_\mu \to A_m \cdot \mu,\ A_o/\varepsilon, \tag{29c}$$

for the spinor propagators

$$[S_F(x - x') = < o \mid \psi(x')\ \psi(x) \mid o >]$$

$$S_F(x - x') \to S_F(x - x') \cdot \left(\frac{H}{\varepsilon\, T^{1/2}}\right), \tag{30}$$

for the boson propagators

$$[\ X_1\ (x - x') \quad = \langle o \mid x_\mu\ (x')\ x_\nu\ (x) \mid o \rangle\ ]$$

$$Z_F(x - x') \rightarrow Z_F(x - x') \cdot \left(\frac{H}{T\varepsilon^2}\right) \tag{31a}$$

$$W_F(x - x') \rightarrow W_F(x - x') \cdot \left(\frac{H}{T\varepsilon^2}\right) \tag{31b}$$

$$A_F(x - x') \rightarrow A_F(x - x') \cdot \left(\frac{H}{T\varepsilon^2}\right) \tag{31c}$$

These are evaluated using the usual techniques [32, 40, 44]. It is simply a matter of counting the $TH\varepsilon\mu$ coefficients as factors in the equations.

The coupling inserted at each vertex of a graph becomes

$$g \rightarrow g\left(\frac{H}{T}\right)^{1/4} \cdot \frac{1}{\sqrt{\varepsilon}} \ . \tag{32}$$

The density of the final states becomes

$$\frac{dn}{dE} \rightarrow \frac{dn}{dE} \cdot \left(\frac{H}{T\varepsilon^2}\right) T^{1/2} . \tag{33}$$

VI. EFFECTS OF THE GRAVITATIONAL POTENTIALS ON WEAK PROCESSES: WEAK INTERACTION TESTS OF THE EQUIVALENCE PRINCIPLE

The effects on selected experiments to test the equivalence principle for the weak interactions can, at last, be computed. Generally these are not suitable since gravitation is extremely weak and the intrinsic signal in a weak-interaction experiment is already usually very weak. The combination of the two factors does not encourage optimism. There are, however, three situations which are of interest. They involve measuring

weak interaction processes indirectly through electromagnetic radiation or the strong interaction.

The first of these is K-capture [45] by the Fe^{55} nucleus with emission of an X-ray. The half-life is short (2.9 yr) [46], so there is a copious number of X-rays from a sample. Their average energy is approximately 6keV; therefore, they can be very efficiently counted with proportional counters or as an average current from a photodiode. In fact, the count rate for a typical source is 10^8/s, currently just beyond the capability of modern counters. The average photodiode current method is therefore the immediately feasible method. The problems of that experiment per se will be left to a later paper. The graph for the process is given in Figure 10.

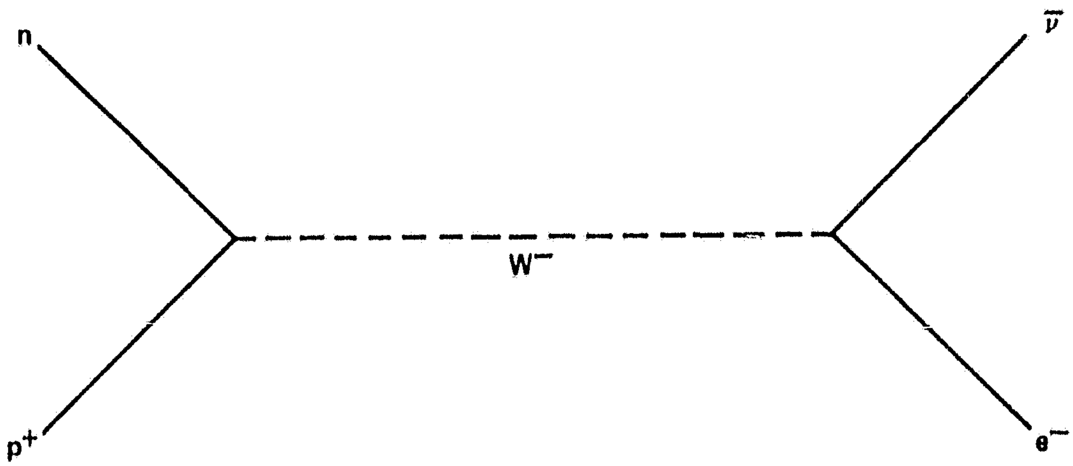

Figure 10. Graph in Figure 9 for K-capture mediated either by W^- emission by electron and absorption by proton or W^+ emission by proton and absorption by electron. It is given just to show where free fields and propagators belong in the amplitude without the additions of Figure 9.

The second case involves ordinary β decay in a measurement suggested by Parker [47]. It involves radiative series which contain β- and α- decay branches. One measures the β rate by counting the α particles in two channels and comparing. Such a branching is illustrated in Figure 11. The α particles can be counted efficiently. Here one increases the signal-to-noise ratio (S/N) by increasing the number of samples and counters so that $S/N \rightarrow U/c^2$, the Newtonian potential. The graph for the process is given in Figure 8.

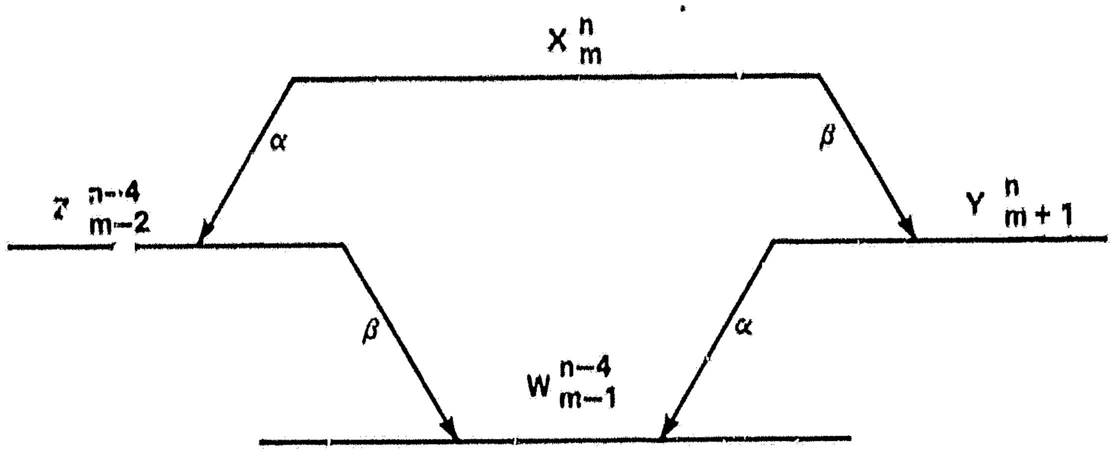

Figure 11. Illustration of typical $\beta - \alpha/\alpha - \beta$ decay $4n + m$ radioactive series with primary parent nucleus X of atomic number n with m protons decaying finally to a stable element W of atomic number $n - 4$ with $m - 1$ protons via nuclei Z and Y.

The last case involves parity nonconserving transitions in heavy atoms mediated by Z_μ boson exchange [48]. The graph is the same as Figure 2 with the γ propagator replaced by the Z_μ propagator. The process occurs for much the same reason as electron capture: In a large atom (high A) the electron wave function has a large value inside the nucleus so that it is close enough to the nucleons to interact via neutral Z_μ boson exchange. The parity nonconserving transitions show in a circular dichroism in selected atomic vapors, notably in thallium and bismuth [48].

The decay constant λ in beta decay is given by Fermi's golden rule,

$$\lambda = \frac{2\pi}{\hbar} |H_{if}|^2 \frac{dn}{dE} , \tag{34}$$

and the excited nuclei then depopulate the excited initial state i into a number of final states f according to the well-known exponential formula

$$N = N_i e^{-\lambda t} . \tag{35}$$

For beta decay and K-capture the amplitude H_{if} for high and low energies is proportional to

$$H_{if} = g^2 \int d^3V \ \bar{n} \gamma^{\mu} p \ W_{\mu\nu} (x - x') \ \bar{\nu} \gamma^{\nu} e. \tag{36}$$

Inserting the T, H, ε, μ factors from equations (28) through (32) and a factor H^{-3} for the integration of the interaction volume V, we find

$$H_{if} \to H_{if} \cdot \left(\frac{H^4}{\varepsilon^9 T^4}\right)\left(\frac{H}{T}\right)^{1/2}. \tag{37}$$

Inserting this into equation (34) with equation (33), we find

$$\lambda \to \lambda \left(\frac{H^5}{\varepsilon^{11} T^5}\right) \left(\frac{H}{T}\right)^{1/2} T^{1/2}, \tag{38}$$

and thus the decay formula [equation (35)] becomes

$$N = N_i \exp \left[- \left(\frac{H^5}{\varepsilon^{11} T^5}\right)\left(\frac{H}{T}\right)^{1/2} T^{1/2} \lambda t \right]. \tag{39}$$

Note that we have neglected "weak magnetism" here. It will be treated in a later report, with the exception of a few remarks which follow. When we impose the metric meshing condition $\varepsilon = \mu = (H/T)^{1/2}$, we find

$$\lambda \to \lambda \ T^{1/2}. \tag{40}$$

If we compare λ at two different altitudes, we find

$$\Delta\lambda = \lambda_1 - \lambda_2 = \lambda \ (T_1^{1/2} - T_2^{1/2}); \tag{41}$$

or, on using equation (6a),

$$\Delta\lambda = \alpha\lambda\,(U_1 - U_2) = \alpha\lambda\,\Delta U. \tag{42}$$

This is precisely the form Nordvedt gives for the metric theories [35].

In the parity-nonconserving atomic transition experiment we use the results of Neuffer and Commins [49]. They use an effective Hamiltonian matrix element

$$< \psi_1 \mid H_{PN} \mid \psi_2 > = \left| -\frac{GQ_W}{2\sqrt{2}}\,\psi_1{}^{*}\,\gamma_5\,\psi_2 \right| \tag{43}$$

using the electronic wave functions ψ_1 and ψ_2. The circular dichroism δ (rotation of plane of linear polarization, or preferential absorption of one circular polarization more than another) is given by

$$\delta = [\,2M \cdot \mathrm{Im}\,(\mathscr{E}_{PN})]\Big/\left[|M|^2 + \frac{2}{3}\,|<\mathscr{E}_2>|^2\right] \tag{44}$$

where M is the magnetic dipole transition probability between two states (say, $7P_{3/2}$ - $7P_{1/2}$ in thallium), $\mathscr{E}_2$ the electron dipole transition between the same states, and $\mathscr{E}_{PN}$ the parity-nonconserving transition probability between the same two states. The sum of M1 and E2 transition matrix contributions is

$$<T> = < P_{3/2} \mid \vec{\mu} \cdot (\hat{x} \times \epsilon) + e\,\hat{\epsilon} \cdot \vec{r} + ie(\hat{\epsilon} \cdot \vec{r})\,(\vec{k} \cdot \vec{r}) \mid P_{1/2} > \tag{45}$$

where $\vec{\mu} = (e\hbar/2mc)\,(\vec{L} + \vec{S})$ and $\hat{\epsilon} = \hat{y}\cos\theta + \hat{z}\sin\theta$, $\vec{r}$ and $\vec{k}$ are the radius vector and wave vector. Using relativistic wave functions, the effects of T, H, ϵ, and μ can be inserted into equations (43) and (45) directly from Will [25]. We find

$$M \to \left(\frac{H}{T\epsilon^2}\right)\left(\frac{\mu}{\epsilon}\right)T^{1/2} \cdot M, \tag{46a}$$

$$\mathcal{E}_2 \to \left(\frac{H}{T\varepsilon^2}\right) T^{1/2} \cdot \mathcal{E}_2, \tag{46b}$$

and

$$\mathcal{E}_{PN} \to \frac{H}{T\varepsilon^2} \quad T^{1/2} \cdot \mathcal{E}_{PN}. \tag{46c}$$

Inserting equations (46) into the circular dichroism formula of equation (44), we find, after some algebra,

$$\delta = \left(\frac{\varepsilon}{\mu}\right) [2M \cdot \operatorname{Im} \mathcal{E}_{PN}] \Bigg] \left[M^2 + \left(\frac{T\varepsilon^4}{H\mu^2}\right) T^{1/2} \mathcal{E}_2^2 \right] . \tag{47}$$

Now in the chosen transition $\mathcal{E}_2^2 >> M^2$, so we expand the denominator and find

$$\delta = \left(\frac{2M \cdot \operatorname{Im} \mathcal{E}_{PN}}{M^2 + \mathcal{E}_2^2}\right)\left(\frac{\varepsilon}{\mu}\right) \left[1 + \left(\frac{\varepsilon^4 T^{3/2}}{H\mu^2} - 1\right) \frac{\mathcal{E}_2}{(M^2 + \mathcal{E}_2^2)}\right]^{-1} . \tag{48}$$

Since $M^2 << \mathcal{E}_2^2$, equation (48) can finally be rewritten

$$\delta = \delta_o \left(\frac{\varepsilon}{\mu}\right) \left[1 - \left(\frac{\varepsilon^4 T}{H\mu^2}\right) T^{-1/2}\right] \tag{49}$$

where δ_o is the value of δ in flat space. If we apply the metric-meshing condition, equation (49) reads

$$\delta = \delta_o \; [1 - T^{-1/2}]. \tag{50}$$

Using equation (6a) again, we find that in a metric theory of gravity the circular dichroism varies as

$$\Delta\delta = \delta_1 - \delta_2 = \alpha\,\delta_0\,(U_1 - U_2) = \alpha\,\delta_0\,\Delta U. \tag{51}$$

Equation (51) is an interesting result.

Because there are no data as yet at hand to describe these experiments, there appears to be little point at the moment in expanding equation (38) or equation (49) in terms of the power series in U listed in equations (6). It is a straightforward exercise if the reader wishes to do so anyway. Rather, we will turn our attention to the possibility of doing the experiments.

VII. THE POSSIBILITY OF ACTUALLY DOING THESE WEAK INTERACTION EQUIVALENCE PRINCIPLE EXPERIMENTS

In the experiments chosen in the preceding section, use was made of the fact that weak interactions are accompanied by other interactions that provide a strong signal. The beta decay signal is quite weak, but X-rays, alpha particles, and 293-nm photons give strong signals. Therefore, we have chosen indirect means to monitor weak interactions. We are, of course, assuming that in the emission of electromagnetic radiation and of alpha-particle decays both the electromagnetic and strong interactions obey the equivalence principle and hence can be used to read the weak interaction's obedience to the equivalence principle. In what follows we give only a brief, preliminary and cursory discussion of possible experiments, deferring detailed analysis to a future report.

The gravitational signals, U/c^2, of the Earth, Jupiter, and the Sun are about 10^{-10}, 10^{-8}, and 10^{-7}. Therefore, any experiment can only allow inaccuracies as small as those numbers if meaningful results are to be obtained. Thus, the success of these experiments depends on strength of signal, detector resolution, numbers of emitted particles, and recovery rate of the detector between counts. Fe^{55} has a half-life of 2.9 yr [46]. A sample would have an emission rate of 10^8/s. The best that can be done for counting X-rays with current off-the-shelf technology is 10^6 - 10^7/s. Therefore, one would have instead to measure the

average current at a photodiode until X-ray proportional counter technology improves (it should eventually). One needs then a photodiode which can maintain a current with a stability and accuracy of 10^{-9} for a solar experiment and 10^{-12} for an Earth orbit or sounding rocket experiment. This may be attainable in the near future.

In the case of alpha-decay in a 4 nucleotide series, one possible chain starts with Thorium in Th^{227} with a half-life of 1.9 yr [46]. It will therefore survive long enough after preparation for use in a satellite or Shuttle-borne payload. It decays to Po^{216}, which leads to a series of rapid alpha and beta decays. The scheme is shown in Figure 12. The energies of different alpha particles are listed in Table 1. They have extremely narrow width (46). Thus, their count rates provide a measure of the intervening beta decay rates ^{216}Po - ^{216}At and ^{212}Bi - ^{212}Po. The question of accuracy depends on the count rate and here at best is 10^{-6}. But with a large number of counting experiments run simultaneously, it can be brought to 10^{-8}. Thus, such an experiment could be done in a solar satellite probe. Again, improvements in technology will probably increase the accuracy of such an experiment so it will become feasible for Earth orbit.

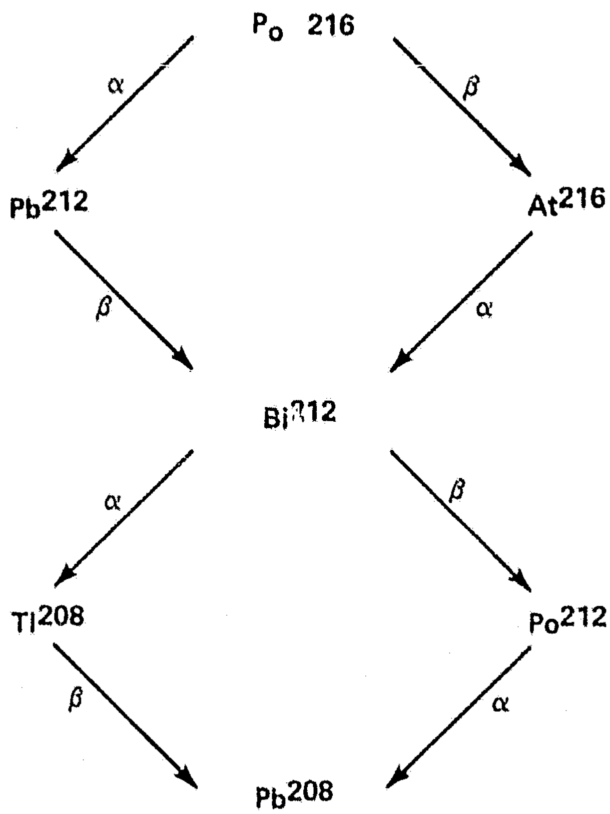

Figure 12. Double β-α/α-β decay scheme for Po^{216} through Bi^{212} down to stable Pb^{208}. Energies are listed in Table 1.

TABLE 1. ENERGIES OF THE ALPHA PARTICLES EMITTED IN THE DECAY SCHEME DESCRIBED IN FIGURE 12

Transition	Energy (MeV)
Po^{216} - Pb^{212}	6.77
At^{216} - Bi^{212}	7.79
Bi^{212} - Tl^{208}	6.05, 6.09
Po^{212} - Pb^{208}	8.78

For the circular dichroism experiment it would seem better to do differential absorption spectroscopy experiments rather than the ones that have been done to test for the existence of the effect. Modern spectrophotometry, under ideal conditions can reach an accuracy of 10^{-12}. Of course, ideal Earth laboratory conditions are not quite like conditions on a rocket payload, but we suggest that such accuracy may be attainable. The hope is that the two circularly polarized beams might be separated and individually have their intensities measured by a photomultiplier. Then the intensity would be individually measured after each beam has passed through the thallium or bismuth (for example) cells. These measurements would be made repeatedly, thus determining the relative absorption in both right and left circularly polarized photons. The difference in their absorptions will give the circular dichroism. It should be possible to develop a method of doing this experiment ultimately at an accuracy of 10^{-12} or better.

VIII. CONCLUSION: DISCUSSION AND OUTLINE OF FURTHER INVESTIGATIONS OF THE WEAK AND STRONG INTERACTIONS AND THE EQUIVALENCE PRINCIPLE

One, of course, likes to see as many different experiments as possible to test our theoretical ideas. We, therefore, see that other aspects of the weak and electroweak interactions should, or could, be tested for equivalence principle violations.

One such aspect is the so-called "weak magnetism" in the Gamow-Teller weak interactions [50]. Holstein has studied the experimental consequences of these interactions in a thorough series of papers [51]. The main result is the appearance of β-α and β-γ correlations in the decays of nuclei in an isotropic triplet [52]. The triplet consists of a central nucleus which emits either γ or two α particles and a β^- and β^+

emitter on either side of it in the periodic table. Two such possible triplets are shown in Figures 13 and 14. Such processes are being studied experimentally for weak magnetism and second-class current effects [53]. The possibility may exist that, if the equivalence principle is violated in the weak interactions, then the β-α and/or β-γ correlations might vary. The calculation is beyond the scope of this current work but will be investigated and reported in a future report. It would require a cyclotron in the payload; but with current efforts to miniaturize superconducting magnets [54], it may someday be possible. For such a test to be worthwhile, the variations in the correlations should be a function of T, H, ϵ, and μ and should probably not exist if gravitation is described by a metric theory.

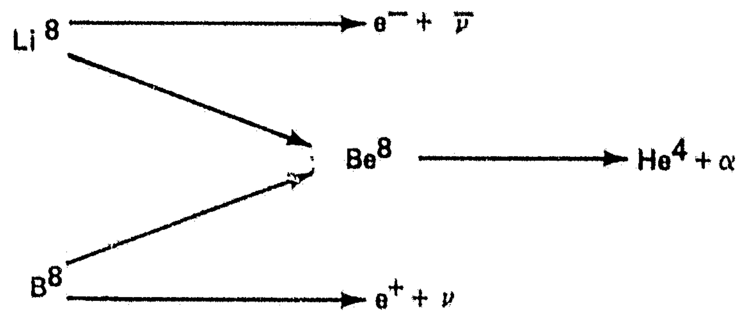

Figure 13. Mirror $\beta^{\pm}$ decay system for Li^8 and B^8 to Be^8 followed by $Be^8 \rightarrow He^4 + \alpha$ for the isotopic triplet Li^8, Be^8, and B^8.

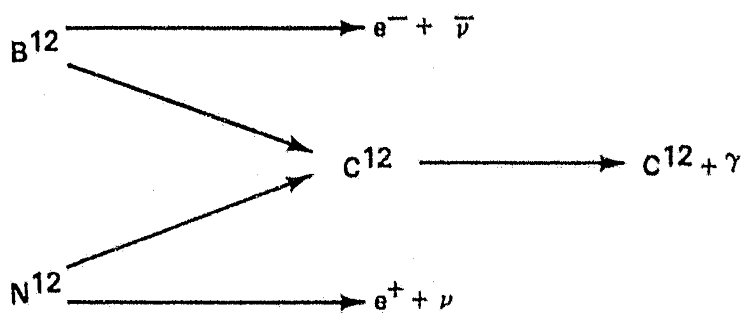

Figure 14. Mirror $\beta^{\pm}$ decay system for B^{12} and N^{12} to C^{12} followed by gamma emission for the isotopic triplet B^{12}, C^{12}, N^{12}.

Another direction the investigation might take would be from decay rate clocks to Eötvös experiments. One would calculate weak self-energies of nucleons and weak binding energies of nuclei using the ideas expressed in this report to include $TH\epsilon\mu$ effects in such calculations. This would clarify the relationship between Eötvös experiments and clock experiments. It should set new limits on ϵ and μ and allow for another aspect of weak interactions to be subjected to such tests. The potentials T, H, ϵ, μ would be fully expanded as power series in U and then all electroweak experiments compared.

One might also hope, as an amusing exercise if nothing else, to generate semiclassical equations of motion for weak-charged particles and their response to the weak field. The idea would be to generate tensor virial relations for weakly interacting particles. The "weak magnetism" binding energy could then be calculated and compared to the post-post-Coulombian nuclear magnetostatic binding energy to see what additional limits, if any, might be placed on ϵ and μ.

Following the ambitious programs outlined previously, the investigation should be extendable to the strong interactions via the grand unified SU(5) in the vector-gluon/quark model. There may be some way to adapt the formalism to the nuclear-pion exchange model to study equivalence principle violations of the low- and medium-energy strong interactions. Experiments that come to mind to be analyzed for feasibility in the formalism are: (1) alpha decay, (2) neutron emission, (3) neutron capture, and perhaps (4) fission. The point made earlier is reiterated: At least at the time of the big bang, when all interactions had the same strength and were, in fact, identical under a symmetry such as SU(5), the formalism certainly was applicable to all interactions. As symmetries are broken and interactions separate and have their couplings assume different values, the potentials $TH\epsilon\mu$ in the Lagrangian remain there in their original form. So the extension appears quite natural. One wonders about the equivalence principle in light of some tantalizing ideas expressed by Weinberg [43] in his Nobel lecture concerning the origin of gravitation as: The remnant of a very complex multiplet boson field, which survives from the big bang only as a coherent average of its isotopic space components, in which this averaging process prevents renormalization so the final interaction is not renormalizable. But back in the big bang when gravitation existed as the fully renormalizable multiplet field that it is, all interactions were equivalent. Again, the almost metaphysical statement is made: Perhaps the principle of equivalence exists amid the chaotic spectrum of different interactions in the present universe because all interactions were equivalent amid the chaotic geometry (determined by a multiplet boson gravitation) in the early universe.

At any rate, this seems to offer a first step toward another generation of equivalence-principle experiments for all interactions, and in as many types of experiments as possible to conceive, estimate theoretically, and then do.

REFERENCES

1. Bertotti, B., (Ed.): Experimental Gravitation, "Enrico Fermi" Course 56, Academic, New York (1974).

2. Davies, R. W., (Ed.): Proceedings of the Conference on Experimental Tests of Gravitational Theories. NASA-JPL Technical Memorandum 33-499, 1971.

3. Shapiro, I. I., Pettengill, G. H., Ash, M. E., Ingalls, R. P., Campbell, D. B. and Dyce, R. B.: Phys. Rev. Lett., 28, 1594, 1972; Morrison, L. V. and Ward, G. C.: Mon. Not. Roy. Astron. Soc., 173, 183, 1975; Dicke, R. H.: Science, 184, 419, 1974; Hill, H. A., Clayton, P. D., Patz, D. L., Healy, A. W., Stebbins, R. T., Oleson, J. R., and Zanoni, C. A.: Phys. Rev. Lett., 33, 1497, 1974. For a theoretical review see Duff, M. J.: Gen. Rel. Grav., 5, 441, 1974.

4. Texas Mauritanian Eclipse Team: Astron. J., 81, 452, 1976.

5. Shapiro, I. I.: Phys. Rev. Lett., 13, 789, 1964; Shapiro, I. I., Ash, M. E., Ingalls, R. P., Smith, W. B., Campbell, D. B., Dyce, R. B., Jurgens, R. F., and Pettengill, G. H.: Phys. Rev. Lett., 26, 1132, 1971; Reasenberg, R. D., Shapiro, I. I., MacNeil, P. E., Goldstein, R. B., Breidenthal, J. C., Brenkle, J. P., Cain, D. L., Kaufman, T. M., Komarek, T. A., and Zygielbaum, A. J.: Astrophys. J., 234, L219, 1979.

6. Fomalont, E. B. and Srame., R. A.: Astrophys. J., 199, 749, 1975; Phys. Rev. Lett., 36, 1475, 1976.

7. Pound, R. V. and Rebka, G. A., Jr.: Phys. Rev. Lett., 4, 337, 1960; Pound, R. V. and Snider, J. L.: Phys. Rev., 140, 788, 1965.

8. Dicke, R. H.: Experimental Relativity, in Relativity, Groups, and Topology, C. DeWitt and B. DeWitt (Ed.), Gordon and Breach, New York, pp. 165-313; Braginsky, V. B. and Ginzburg, V. L.: Soviet Phys.-Dokl., 19, 290, 1974.

9. Weber, J.: Phys. Rev. Lett., 25, 180, 1970; Nuovo. Cim., 4B, 197, 1972; Gravitation Radiation Experiments, in Ref. 1, pp. 263-279.

10. Douglass, D. H. and Braginsky, V. B.: Gravitational Radiation Experiments, in General Relativity: An Einstein Centenary Survey, S.W. Hawking and W. Israel (Ed.), Cambridge U.P., 1979, pp. 90-137.

11. See, for example, Paik, H.: Low Temperature Gravitational Wave Detector, in Ref. 1, pp. 515-524.

12. Caves, C. M., Thorne, K. S., Drever, R. P., Sandberg, V. D., and Zimmerman, M.: Rev. Mod. Phys., 52, 344, 1980.

13. Braginsky, V. B., Vorontsov, Y. J., and Thorne, K. S.: Science, 209, 547, 1980.

14. Unruh, W. G.: Phys. Rev. D. 19, 2888, 1979.

15. Vessot, R. F. C. and Levine, M. W.: Gen. Rel. Grav., 10, 181, 1979.

16. Anderson, J. D., Esposito, P. B., Martin, W., Thornton, C. L., and Muhleman, D. O.: Astrophys. J., 200, 221, 1975; also see Reasenberg, R. D., et al., in Ref. 5.

17. Van Patten, R. A. and Everitt, E. W. F.: Phys. Rev. Lett., 36, 629, 1976; Acta Astronautica, 5, 77, 1978; Celestial Mechanics, 13, 429, 1976.

18. Thorne, K. S. and Braginsky, V. B.: Astrophys. J., 204, L1, 1976, Estabrook, F. B. and Wahlquist, H. D.: Gen. Rel. Grav., 6, 439, 1975; Acta Astronautica 5, 5, 1978. Satellite orbital frame dragging and gravitational radiation are linked together in Mashhoon, B.: Astrophys. J., 227, 1019, 1979.

19. Everitt, C. W. F.: The Gyroscope Experiment-I: General Description and Analysis of Gyroscope Performance, in Ref. 1, pp. 331-360; and see other papers following it in Ref. 1.

20. Will, C. M.: The Theoretical Tools of Experimental Gravitation, in Ref. 1, pp. 1-110; The Confrontation between Gravitation Theory and Experiment, in Ref. 10, pp. 24-89. Thorne, K. S., Will, C. M., and Ni, W. T.: Theoretical Frameworks for Testing Relativistic Gravity - A Review, in Ref. 2, pp. 10-33.

21. Thorne, K. S. and Will, C. M.: Astrophys. J., 163, 595, 1971. Will, C. M.: Astrophys. J., 163, 611, 1971, ibid. 169, 125, 1971. Ni, W. T.: ibid, 176, 769, 1972. Lee, D. L., Caves, C. M., Ni, W. T., and Will, C. M.: ibid., 206, 555, 1976.

22. Nordvedt, K., Jr.: Phys. Rev., 169, 1014, 1968; ibid., 169, 1017, 1968; ibid., 180, 1293, 1969; ibid., D 3, 1683, 1971.

23. Thorne, K. S., Lee, D. L., and Lightman, A. P.: Phys. Rev. D7, 3563, 1973.

24. Lightman, A. P. and Lee, D. L.: Phys. Rev. D 8, 364, 1973. For a preliminary study that led to the formalism see Lee, D. L and Lightman, A. P.: ibid., 7, 3578, 1973.

25. Will, C. M.: Phys. Rev., D 10, 2330, 1974.

26. Haugan, M. P. and Will, C. M.: Phys. Rev. D, 15, 2711, 1977.

27. Fennelly, A. J.: unpublished colloquium given at Vanderbilt University, December 1978.

28. Fennelly, A. J. and Bellis, R. E.: paper in preparation, 1980. A preliminary proposal of the NMR clock experiment has been submitted to NASA, 1980.

29. Brecher, K.: Astrophys. J., 219, L117, 1978.

30. Nordvedt, K.: unpublished colloquium at the Marshall Space Flight Center, August 1978, and report to MSFC, Study Report: Development of a Precision Decay Clock, unpublished, November 1978.

31. In such experiments we do not directly measure the weak interaction, rather we measure a secondary or by-product interaction, such as a subsequent emission of an alpha particle by the daughter nucleus or a K-capture X-ray. These give a strong signal and are easily counted. Thus we will describe experiments which test the equivalence principle and the weak interaction modulo the electromagnetic or strong interaction.

32. Weinberg, S.: Phys. Rev. Lett. 19, 1264, 1967; ibid., 27, 1688, 1971; ibid., 29, 388, 1972. Hsu, J. P.: Phys. Rev. Lett. 36, 646, 1976. Weinberg, S.: Rev. Mod. Phys. 46, 255, 1974; Bernstein, J.: ibid., 46, 7, 1974.

33. Haugan, M. P. and Will, C. M.: Phys. Rev. Lett. 37, 1, 1976.

34. Hsu, J. P.: Phys. Rev., D 17, 3164, 1978.

35. Nordvedt, K., Phys. Rev., D 11, 245, 1975.

36. Plebanski, J. P.: Phys. Rev. 118, 1396, 1960.

37. Volkov, A. M., Izmest'ev, A. A., and Skrotskii, G. V.: Soviet Phys.-JETP 32, 686, 1971.

38. Mashhoon, B.: Phys. Rev. D 7, 2807, 1973; ibid., 8, 4297, 1973; ibid., 10, 1059, 1974; ibid., 1, 2679, 1975.

39. Haugan, M. P.: Ann. Phys (N.Y.), 118, 156, 1979.

40. Bjorken, J. D. and Drell, S. D.: Relativistic Quantum Mechanics, McGraw-Hill, New York, 1964; Relativistic Quantum Fields, McGraw-Hill, New York, 1965.

41. Brodsky, S. J.: Quantum Electrodynamics and the Theory of the Hydrogen Atom, in Atomic Physics and Astrophysics, M. Chretien and E. Lipworth (Ed.), Brandeis University Summer Institute in Theoretical Physics, 1969, Volume 1, Gordon and Breach, New York, 1971, pp. 95-174.

42. Abers, E. S. and Lee, B. W.: Gauge Theories, Phys. Reports 9C (No. 1), pp 1-141, 1973.

43. Weinberg, S.: Rev. Mod. Phys., 52, 515, 1980; A. Salam, ibid., 525; Glashow, S. L.: ibid., 539.

44. Schecter, J. and Ueda, Y.: Phys. Rev., D 2, 736, 1970; Salomonson, P. and Ueda, Y.: ibid, 11, 2606, 1975. These authors exhibit the complete counterterm Langrangian for renormalization.

45. Bambynek, W., Behrens, H., Chen, M. H., Crasemann, B., Fitzpatrick, M. L., Ledingham, K. W. D., Genz, H., Mutterer, M., and Intemann, R. L.: Rev. Mod. Phys., 49, 77, 1977.

46. Leighton, R. B.: Principles of Modern Physics, McGraw-Hill, New York, 1959.

47. P. Parker has proposed this idea to NASA and is now developing it for possible flight. I am indebted to Dr. P. Eby for discussions of this idea and his encouragement to consider it here.

48. Bankov, L. M. and Zolotorev, M. S.: JETP Lett. 27, 357, 1978; Conti, R., Buchsbaum, P., Chu, S., Commins, E., and Hunter, L.: Phys. Rev. Lett. 42, 343, 1979; Bouchiat, M..A., Bouchiat, C. C.: Phys. Lett. 48B, 111, 1974; Henley, E. M. and Wilets, L.: Phys. Rev. A 14, 1411, 1976; Neuffer, D. V. and Commins, E. D.: ibid. 16, 844, 1977; Chu, S., Commins, F. D., and Conte, R.: Phys. Lett. 60A, 96, 1977. An interesting alternative view is given in: Epstein, G. N.: Phys. Rev. Lett. 44, 905, 1980.

49. Neuffer, D. V. and Commins, E. D.: Phys. Rev. A 16, 844, 1977, as cited in Ref. 48 above.

50. Feynman, R. P., and Gell-Mann, M.: Phys. Rev. 109, 143, 1958; Weinberg, S.: ibid. 112, 1375, 1958.

51. Holstein, B. R. and Treiman, S. B.: Phys. Rev. C 3, 1921, 1971. Holstein, B. R.: ibid. 4, 764, 1971; ibid. 5, 1529, 1972; Rev. Mod. Phys. 46, 789, 1974.

52. Holstein, B. R.: Phys. Rev. C 4, 740, 1971.

53. Tribble, R. E. and Garvey, G. T.: Phys. Rev. C, 12, 967, 1975; Bowles, T. J. and Garvey, G. T.: Phys. Rev. C, 18, 1447, 1978; Brandle, H., Miklos, G., Roesch, L. Ph., Telegdi, V. L., Truttmann, P., Zehnder, A., Grenacs, L., Lebrunn, P., and Lehmann, J.: Phys. Rev. Lett. 41, 299, 1978; McKeown, R. D., Garvey, G. T., and Gagliardi, C. A.: Argonne National Laboratory Reprint, 1980.

54. NASA Space Magnetism and Cryogenics Report: D. B. Sullivan (Ed.), NASA-NBS Study, The Role of Superconductivity in the Space Program: An Assessment of Present Capabilities and Future Potential, NBSIR 78-885, National Bureau of Standards, Boulder, CO, 1978.

APPROVAL

ADAPTATION OF THE THεμ FORMALISM FOR THE ANALYSIS OF THE EQUIVALENCE PRINCIPLE IN THE PRESENCE OF THE WEAK AND ELECTROWEAK INTERACTION

By A. J. Fennelly

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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